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WAVES CAUSED BY MOVING LOADS IN AN ISOTROPIC LAYER INHOMOGENEOUS THROUGH THE THICKNESS*

A.V. BELOKON and A.V. NASEDKIN

Problems on motion with a constant subseismic velocity of an oscillating load on the boundary of an isotropic elastic layer inhomogeneous over the thickness are studied in a three-dimensional formulation. Quantitative estimates are given for the upper limits on the magnitudes of the velocity of motion and the load vibration frequency for which a unique solution exists for the problem in energy classes. In cases when no energy solution exists, principles are formulated to extract the unique solution and a solution is given in the far field. Results are presented of numerical computations of the wave field characteristics in the case of the motion of a normal concentrated load in a homogeneous layer. Situations are noted in which a different number of waves propagates in different layer domains. The problems considered are of interest for seismology and in designing aerodrome coverings.

1. Let an elastic, isotropic, medium inhomogeneous through the thickness occupy a domain $\Pi = (-\infty < x, y < +\infty; 0 \leqslant z \leqslant 1)$, whose lower boundary is clamped, i.e.,

$$\mathbf{u}(x, y, 0, t) = 0 \tag{1.1}$$

The upper boundary is loaded in such a manner that in a moving system of coordinates

$$x_1 = x - wt,\ x_2 = y,\ x_3 = z,\ t = t \tag{1.2}$$

the boundary conditions have the form

$$\sigma^{3j}(x_1, x_2, 1, t) = f^j(x_1, x_2)e^{i\Omega t}\ (x_1, x_2) \in S;\ j = 1, 2, 3 \tag{1.3}$$

Here S is a bounded domain whose boundary has a continuously differentiable curvature at each point. It is assumed that the layer surface is not loaded outside S while the elastic constants of the material are subject to the conditions

$$\lambda(x_3),\ \mu(x_3) \in C_1(0, 1),\ \rho(x_3) \in C(0, 1);\ \lambda, \mu, \rho \geqslant \delta_* > 0$$

We will study the steady motion in the moving coordinate system (1.2) by assuming that the displacement vector in (1.2) has the form

$$\mathbf{u}(x - wt, y, z, t) = \mathbf{v}(x_1, x_2, x_3)e^{i\Omega t} \tag{1.4}$$

In this case the system of equations of the theory of elasticity takes the form (δ_{kj} is the Kronecker delta)

$$(\lambda v_{m,m})_{,j}\delta_{kj} + (\mu(v_{k,j} + v_{j,k}))_{,j} - \rho w^2 v_{k,11} = -\rho\Omega^2 v_k - 2i\rho\Omega w v_{k,1},\quad k = 1, 2, 3 \tag{1.5}$$

For convenience, the problem will be formulated in dimensionless form. The change to dimensional parameters is made by means of the formulas

$$x = \frac{x_d}{h},\ y = \frac{y_d}{h},\ z = \frac{z_d}{h},\ w = \frac{w_d}{c_{20d}},\ \Omega = \Omega_d\frac{h}{c_{20d}},\ t = t_d\frac{c_{20d}}{h}$$

$$\mu = \frac{\mu_d}{\mu_{0d}},\ \lambda = \frac{\lambda_d}{\mu_{0d}},\ \sigma^{ij} = \frac{\sigma_d^{ij}}{\mu_{0d}},\ f^j = \frac{f_d^j}{\mu_{0d}},\ \rho = \frac{\rho_d}{\rho_{0d}}$$

$$u_k = \frac{u_{kd}}{h},\quad v_k = \frac{v_{kd}}{h}$$

$$\rho_{0d} = \max\rho_d(z_d),\ \mu_{0d} = \min\mu_d(z_d),\ 0 \leqslant z_d \leqslant h;\ c_{20d} = \sqrt{\mu_{0d}/\rho_{0d}}$$

where the dimensional quantities (except for the layer thickness h) are marked with the subscript d.

We call the problem formulated for Ω, $w \neq 0$ problem C. The case $\Omega = 0$, $w \neq 0$ corresponds

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to problem B, and the case $\Omega \neq 0,\ w = 0$ problem A. For convenience in the subsequent discussions, we shall write the symbol ω in place of Ω in (1.4) for this last problem.

Problems C and B were studied for a half-space in /1, 2/. Problems A-C were investigated in a three-dimensional formulation for a fluid stratified in depth /3/ and an elastic half-space* (*Babeshko, V.A., Glushkov E.V. and Glushkova N.V., Elastic wave excitation by a moving harmonic source, Krasnodar, 1985. Dep. 6470-85 in VINITI, September 3, 1985.) as well as for an acoustic layer.** (**Belokon A.V. and Nasedkin A.V., A model problem on wave propagation from moving pulsating loads in an elastic layer. Rostov n/D, 1986. Dep. 3359-B86, in VINITI, May 11, 1986.). Questions similar to some of those elucidated below were also examined in /4, 5/ where contact problems for an anisotropic layer were studied.

We first study problems A-C in energy classes.

Definition 1. The vector function $\mathbf{v} \in H_{1\Pi}$ is called the generalized solution of the problems A-C if for any $\varphi \in H_{1\Pi}$ it satisfies the integral identity

$$(\mathbf{v}, \varphi)_{H_{1\Pi}} - \int_\Pi \rho\,(wv_{k,1} - i\Omega v_k)(w\overline{\varphi}_{k,1} + i\Omega\overline{\varphi}_k)\,d\Pi = \int_S f^j(x_1, x_2)\,\overline{\varphi}_j(x_1, x_2, 1)\,dx_1\,dx_2 \tag{1.6}$$

Here $H_{1\Pi}$ is the space of vector functions (real in the case of problem B) that satisfy condition (1.1) and leave a finite norm generated by the scalar product

$$(\mathbf{v}, \varphi)_{H_{1\Pi}} = \int_\Pi [2\mu\varepsilon_{mk}(\mathbf{v})\,\varepsilon_{mk}(\overline{\varphi}) + \lambda v_{k,k}\overline{\varphi}_{j,j}]\,d\Pi$$

Theorem 1. Let

$$f^j(x_1, x_2) \in L_{ps},\ p > {}^4/_3 \tag{1.7}$$

Then constants $m,\ \omega_0$ exist such that when the conditions

$$\Omega < \omega_0,\ w < (1 - \Omega/\omega_0)\sqrt{m/\rho_0};\ \rho_0 = \max_{x_3}\rho(x_3) \tag{1.8}$$

are satisfied an energy solution of the problem A, B, C exists.

The proof of Theorem 1 is by analogy with the proof of Theorem 2 from /6/.

2. For practical purposes it is not only important to establish the existence of m and ω_0 but also to give a quantitative estimate of them. To obtain it we introduce the spaces H_B and H_{B_0}, where the scalar product therein id defined, respectively, by the left side of (1.6) and the expression

$$(\mathbf{v}, \varphi)_{H_{B_0}} = \mu_0 \int_\Pi \left[2\varepsilon_{mk}(\mathbf{v})\,\varepsilon_{mk}(\overline{\varphi}) + \left(\frac{\lambda}{\mu}\right)_0 v_{k,k}\overline{\varphi}_{j,j}\right] d\Pi -$$
$$\rho_0 \int_\Pi (wv_{k,1} - i\Omega v_k)(w\overline{\varphi}_{k,1} + i\Omega\overline{\varphi}_k)\,d\Pi$$
$$\left(\mu_0 = \min_{x_3}\mu(x_3),\ \left(\frac{\lambda}{\mu}\right)_0 = \min_{x_3}\frac{\lambda(x_3)}{\mu(x_3)}\right)$$

Obviously,

$$\|\mathbf{v}\|_{H_B} > \|\mathbf{v}\|_{H_{B_0}} \tag{2.1}$$

Lemma 1. Under the conditions of Theorem 1 the solution of problem C is given by the formula

$$v_k = \frac{1}{4\pi^2}\iint_{-\infty}^{+\infty} \frac{G_k(\alpha, \beta, x_3, w\alpha + \Omega)}{D(\alpha, \beta, w\alpha + \Omega)} \exp\{-i(\alpha x_1 + \beta x_2)\}\,d\alpha\,d\beta \tag{2.2}$$

where $G_k,\ D$ are entire functions of their arguments.

The lemma is proved by the scheme used to prove Theorem 1 from /6/.

For a medium with constant coefficients $\rho_0,\ \mu_0$ and $\lambda_0 = \mu_0(\lambda/\mu)_0$ (problem C_0) the formula for v_k can be constructed explicitly and has the form (2.2) with the functions $G_{0k},\ D_0$ in place of G_k and D, respectively, where

$$D_0(\alpha, \beta, \omega) = \operatorname{ch}\gamma_2 D_1(\alpha, \beta, \omega),\ D_1(\alpha, \beta, \omega) = 4\gamma^2\theta - \tag{2.3}$$
$$(\theta^2 + 4\gamma^4)\operatorname{ch}\gamma_1\operatorname{ch}\gamma_2 + \gamma^2(\theta^2 + 4\gamma_1^2\gamma_2^2)\operatorname{sh}\gamma_1\operatorname{sh}\gamma_2/(\gamma_1\gamma_2)$$
$$\omega = w\alpha + \Omega,\ \gamma^2 = \alpha^2 + \beta^2,\ \theta = 2\gamma^2 - \omega^2$$
$$\gamma_1 = \sqrt{\gamma^2 - \frac{1 - 2\nu_0}{2(1 - \nu_0)}\omega^2},\ \gamma_2 = \sqrt{\gamma^2 - \omega^2},\ \nu_0 = \frac{\lambda_0}{2(\lambda_0 + \mu_0)}$$

and the explicit form of G_{0k} is easily reproduced.

Lemma 2. Let $\Omega = 0$, $w < w_R$ or $0 < \Omega < \pi/2$, $w < w_0(\Omega) < w_R$, where $w_R c_{20d}$ is the velocity of Rayleigh surface wave propagation and $w_0(\Omega)$ is the minimum value between $w_1 = \sqrt{1-(2\Omega/\pi)^2}$ and w for which the system of equations

$$D_1(\alpha, 0, w\alpha+\Omega) = 0, \ \frac{d}{d\alpha} D_1(\alpha, 0, w\alpha+\Omega) = 0 \quad (\alpha \neq -\Omega/w)$$

has the real solution α. Then the dispersion equation of problem C_0

$$D_0(\alpha, \beta, w\alpha+\Omega) = 0 \quad (\alpha \neq -\Omega/w)$$

has no real solutions.

Note that a simpler but also deeper condition ensuring the validity of the last assertion is satisfaction of the inequality $w/v_R + 2\Omega/\pi < 1$.

Lemma 3. Under the conditions of Lemma 2 and (1.7) problem C_0 has a unique energy solution determined by (2.2).

Theorem 2. Under the conditions of Lemma 2 and (1.7) problem C has a unique energy solution defined by (2.2).

Theorem 2 results from Lemmas 2 and 3 and the inequality (2.1) and enables us to give a quantitative estimate of the constants m and ω_0 from (1.8).

3. We will not study the non-energy solutions of problem C. In order to construct a unique solution we invoke the principle of limiting absorption and the principle of energy radiation to analyse the non-energy solution. In the case of plane problems these principles are analysed in detail in /6, 7/. The process of passing from an elastic medium to a medium with absorption is described in sufficient detail in the papers mentioned above. The structure of the construction of the solution of the plane problem C for a medium with absorption is described in /6/. On the basis of these papers it can be shown that the solution has the form (2.2) for a medium with absorption, where the quantity $w\alpha + \Omega - i\varepsilon$ occurs instead of $w\alpha + \Omega$. Therefore, the solution in a medium with absorption has the form

$$v_k^\varepsilon = \frac{1}{4\pi^2} \iint\limits_{-\infty}^{+\infty} \frac{G_k(\alpha, \beta, x_3, w\alpha+\Omega-i\varepsilon)}{D(\alpha, \beta, \omega\alpha+\Omega-i\varepsilon)} \exp\{-i(\alpha x_1 + \beta x_2)\}\, d\alpha\, d\beta \tag{3.1}$$

Lemma 4. The equation

$$D(\alpha, \beta, w\alpha+\Omega-i\varepsilon) = 0 \tag{3.2}$$

has no real solutions (α, β) for $\varepsilon \neq 0$.

The zeros (3.2) obviously determine the homogeneous solutions of the boundary value problem (1.1), (1.5), (1.3) for $f^j = 0$ and $\varepsilon > 0$, and consequently, the homogeneous solutions that do not grow at infinity in a medium with absorption are zeros. Therefore, (3.1) determines the complete and unique solution in a medium with absorption in the class of functions that do not grow at infinity.

On the basis of the principles of limiting absorption, the solution for the elastic medium has the form

$$v_k = \lim_{\varepsilon \to +0} v_k^\varepsilon \tag{3.3}$$

where v_k^ε is given by (3.1).

Let Γ_B, Γ_A denote, respectively, the set of real zeros of the equations

$$D(\alpha, \beta, w\alpha+\Omega) = 0, \ D(\alpha, \beta, \omega) = 0 \tag{3.4}$$

Obviously the set Γ_B contains points of intersection of the set Γ_A with the plane

$$\omega = w\alpha + \Omega \tag{3.5}$$

in the space (α, β, ω).

Lemma 5. Let $\Omega < \infty$, $w < \sqrt{m/\rho_0}$. Then, if the dispersion surfaces Γ_A do not intersect mutually and the plane (3.5) is not tangent to Γ_A, then the set Γ_B is represented in the form of the finite union of N closed curves L_j, $j = 1, 2, \ldots, N$ which have no common points and do not degenerate into a point. Hence

$$|\operatorname{grad} D|^2 = (D_{,\alpha} + wD_{,\omega})^2 + D_{,\beta}^2 \neq 0, \ \forall (\alpha, \beta) \in L_j \tag{3.6}$$

Theorem 3. Under the conditions of Lemma 5 the solution of problem C is given by using the following formula:

$$v_k = \frac{1}{4\pi^2}\ \text{v. p.} \iint\limits_{-\infty}^{+\infty} \frac{G_k(\alpha, \beta, x_3, w\alpha + \Omega)}{D(\alpha, \beta, w\alpha + \Omega)} \exp\{-i(\alpha x_1 + \beta x_2)\}\, d\alpha\, d\beta - \tag{3.7}$$

$$\frac{1}{4\pi i} \sum_{j=1}^{N} \oint\limits_{L_j} R_k(\alpha, \beta, x_3) \exp\{-i(\alpha x_1 + \beta x_2)\}\, ds$$

$$R_k(\alpha, \beta, x_3) = \frac{G_k(\alpha, \beta, x_3, w\alpha + \Omega)}{|\operatorname{grad} D(\alpha, \beta, w\alpha + \Omega)|} \operatorname{sign} D_{,\omega}$$

Theorem 3 is proved on the basis of an analysis of the shift of the real roots (α_0, β_0) of (3.5) when ε is introduced into the complex domain.

Namely, when condition (3.6) is satisfied the roots of (3.2) close to (α_0, β_0) can be represented in the form

$$\alpha = \alpha_0 + \varepsilon\tau\cos\zeta + o(\varepsilon), \quad \beta = \beta_0 + \varepsilon\tau\sin\zeta + o(\varepsilon) \tag{3.8}$$

Expanding the function on the left-hand side of (3.2) in a series in ε in the neighbourhood of (α_0, β_0) we obtain that

$$\tau = -ic_\zeta^{-1}(\alpha_0, \beta_0) \tag{3.9}$$

$$c_\zeta(\alpha_0, \beta_0) = \frac{\partial\omega}{\partial\zeta} - w\cos\zeta, \quad \frac{\partial\omega}{\partial\zeta} = \frac{\partial\omega}{\partial\alpha}\cos\zeta + \frac{\partial\omega}{\partial\beta}\sin\zeta \tag{3.10}$$

where $\partial\omega/\partial\zeta$ is the algebraic projection of the group velocity vector with the components

$$\partial\omega/\partial\alpha = -D_{,\alpha}/D_{,\omega}, \quad \partial\omega/\partial\beta = -D_{,\beta}/D_{,\omega} \tag{3.11}$$

in the direction ζ and evaluated at the point (α_0, β_0) for problem A.

Formulas (3.8) and (3.9) indeed determine the shift of the roots (α_0, β_0) in the complex domain on introducing ε.

If the plane (3.5) is tangent to the dispersion surface of the set Γ_A at the point (α_1, β_1) then for the existence of a bounded limit (3.3), satisfaction of the equalities

$$G_k(\alpha_1, \beta_1, x_3, w\alpha_1 + \Omega) = 0$$

is necessary and sufficient.

4. We assume that the curves L_j $(j = 1, 2, \ldots, N)$ have a curvature different from zero at each point. Later we shall need the formula /8/

$$\int\limits_{\Gamma} f(\alpha, \beta) \exp\{-i(\alpha x_1 + \beta x_2)\}\, ds \approx \sqrt{\frac{2\pi}{r}} \sum_k \frac{f(\alpha_k, \beta_k)}{\sqrt{|\varkappa_k(\theta)|}} \times \tag{4.1}$$

$$\exp\{-irl_k\cos(\theta - \eta_k) - {}^1\!/_4\pi i \operatorname{sign}(\beta''(\alpha_k)\sin\theta)\} + O(r^{-1}),$$

$$r \to \infty; \quad l_k = l(\alpha_k, \beta_k), \quad \alpha_k = l_k\cos\eta_k, \quad \beta_k = l_k\sin\eta_k,$$

$$x_1 = r\cos\theta, \quad x_2 = r\sin\theta$$

Here $\Gamma = \Gamma(\alpha, \beta)$ is a curve in R^2, which in this case agrees with part of the curve L_j, and α_k, β_k are stationary points determined from the equation

$$\frac{d\alpha}{ds}\cos\theta + \frac{d\beta}{ds}\sin\theta = 0, \quad (\alpha, \beta) \in \Gamma \tag{4.2}$$

or, equivalently, from the system of equations

$$D(\alpha, \beta, w\alpha + \Omega) = 0, \; D_{,\alpha} - D_{,\beta}\operatorname{ctg}\theta + wD_{,\omega} = 0 \tag{4.3}$$

for fixed w and Ω; $\varkappa_k(\theta) > 0$ is the curvature of the curve Γ at the point (α_k, β_k) depending on θ by virtue of (4.2) or (4.3).

Using (3.7) and (4.1) it can be shown that the solution of problem C in the far field has the form

$$v_k \approx \frac{i}{\sqrt{2\pi r}} \sum_{j=1}^{N} \sum_{m=1}^{M(j)} \frac{H(c_\theta{}^m)}{\sqrt{|\varkappa_m(\theta)|}} R_k(\alpha_m, \beta_m, x_3) \times \tag{4.4}$$

$$\exp\{-i(l_m r\cos(\theta - \eta_m) - {}^1\!/_4 \operatorname{sign} c_n{}^m)\} + O(r^{-1}), \quad r \to \infty$$

$$c_\theta{}^m = c_\theta(\alpha_m, \beta_m), \quad c_n{}^m = c_n(\alpha_m, \beta_m)$$

Here $H(x)$ is the Heaviside function, (α_m, β_m) $(m = 1, 2, \ldots, M(j))$ is the complete set of stationary points on L_j for a fixed θ, where $M(j) = 2$ in the case $\varkappa > 0$ everwhere on L_j; c_θ, c_n are defined in the same way as (3.10), and n is the external unit normal to L_j.

Asymptotic representations analogous to (4.4) have been obtained earlier /3/ for other problems with moving pulsating effects (see the papers cited in the previous footnotes also).

The solution (4.4) obtained enables us to make an energy analysis of the waves being propagated. To do this, the energy balance equation for the system of equations of motion in the moving coordinate system (1.2) is written in the form

$$\begin{aligned} &\partial \mathrm{E}/\partial t + \operatorname{div} \mathbf{J} = 0 \\ &\mathrm{E} = \tfrac{1}{2}[\sigma_{kl}(\mathbf{u}) u_{k,l} + \rho (u_k^{\cdot} - w u_{k,1})^2] \\ &J_m = -[\sigma_{km}(\mathbf{u})(u_k^{\cdot} - w u_{k,1}) + \delta_{1m} w \mathrm{E}], \quad m = 1, 2, 3 \\ &\mathbf{u}(x_1, x_2, x_3, t) = \mathbf{u}(x - wt, y, z, t) \end{aligned} \tag{4.5}$$

Here E is the mechanical energy in the moving coordinate system. Hence, it is natural to consider J_m as the components of the energy flux density vector. (These energy characteristics are introduced in dimensionless form.)

Taking into account that for large r the boundary conditions of the problem under consideration are homogeneous, we find

$$\frac{\partial \mathrm{E}_c}{\partial t} + \frac{\partial J_{1c}}{\partial x_1} + \frac{\partial J_{\ c}}{\partial x_2} = 0 \quad \left(\mathrm{E}_c = \int_0^1 \mathrm{E}\, dx_3, \quad J_{kc} = \int_0^1 J_k\, dx_3\right) \tag{4.6}$$

Now we represent one of the wave (4.4) in real form

$$u_k = r^{-1/2}(v_{kc} \cos \delta + v_{ks} \sin \delta), \quad \delta = \alpha x_1 + \beta x_2 - \Omega t \tag{4.7}$$

where v_{kc} and v_{ks} are, respectively, the real and imaginary parts of the expression

$$i(2\pi\, |\varkappa\, |)^{-1/2} R_k(\alpha, \beta, x_3) \exp(1/_4\, \pi i \operatorname{sign} c_n)$$

at the stationary points.

Furthermore, we calculate the quantities averaged over the period of vibrations that occur in (4.5) and (4.6). Taking into account that the average energy and the energy flux from (4.6) are asymptotically additive quantities for waves of the form (4.4), and executing operations analogous to /6/ for deriving the average formulas in the plane problems, we find

$$\begin{aligned} &E = \frac{1}{T}\int_0^T \mathrm{E}_c\, dt = \frac{1}{2}(w\alpha + \Omega)^2 B(\mathbf{v}, \mathbf{v}) \\ &T = \frac{2\pi}{\Omega}, \quad B(\mathbf{v}, \mathbf{v}) = \frac{1}{r}\int_0^1 \rho(x_3) \sum_{k=1}^{3} (v_{kc}^2 + v_{ks}^2)\, dx_3 \\ &P_k = \frac{1}{T}\int_0^T J_{kc}\, dt = \frac{1}{2}(w\alpha + \Omega)^2 B(\mathbf{v}, \mathbf{v}) \times \begin{cases} (\partial\omega/\partial\alpha - w), & k = 1 \\ \partial\omega/\partial\beta, & k = 2 \end{cases} \end{aligned} \tag{4.8}$$

Taking into account that at the stationary points

$$\frac{\partial \omega}{\partial \alpha} \sin\theta - \frac{\partial \omega}{\partial \beta} \cos\theta - w \sin\theta = 0$$

as follows from (4.3) and (3.11), it can be obtained from (4.8) that the energy flux in the direction of the r axis for fixed θ equals

$$P_r = (\partial\omega/\partial\theta - w\cos\theta)\, E \tag{4.9}$$

while the energy flux in the direction θ equals zero.

We now formulate the principle of energy radiation for problem C.

Definition 2. We consider the solution of problem C to be subjected to the energy radiation principle if the condition $0 < P_r < \infty$ is satisfied for a wave of the form (4.7) being propagated.

Since only those waves for which $c_\theta^m = \partial\omega/\partial\theta - w\cos\theta > 0$, are actually selected by the functions $H(c_\theta^m)$ in (4.4), it follows from (4.9) that $P_r > 0$. Therefore, the solution constructed on the basis of the limiting absorption pronciple under the conditions of Lemma 5 does not contradict the energy radiation principle. It can be shown that the solution constructed on the basis of the energy radiation principle will have the form (3.7). Therefore the following will hold.

Theorem 4. Under the conditions of Lemma 5 the solution of problem C constructed on the basis of the limiting absorption and the energy radiation principles are in agreement.

It can be obtained from (4.7)-(4.9) that the total energy flux passing through an infinitely remote cylindrical surface equals

$$P = \int_0^{2\pi} r P_r\, d\theta = \frac{1}{4\pi} \sum_{j=1}^{N} \oint_{L_j} \int_0^1 \rho \sum_{k=1}^{3} |G_k(\alpha, \beta, x_3, w\alpha + \Omega)|^2\, dx_3 \frac{(w\alpha + \Omega)^2}{|D_{,\omega} \operatorname{grad} D|}\, ds$$

We note that the criterion for extracting a unique solution of problem C cannot be the condition $P > 0$ since it is clear from the preceding that a non-unique solution can be constructed in this case.

5. As an illustration of the wave propagation processes in the far field we consider problem C for a homogeneous layer subjected to the moving pulsating normal concentrated load $f^3(x_1, x_2) = \delta(x_1, x_2)$. The values of Ω and w are selected so that the conditions of Lemma 5 are satisfied.

The characteristic form of the curves L_j is shown in Figs.1-3 for $\beta > 0$ in this case for the following values of w, Ω, v_0: 0.5, 2, 0.29 (Fig.1), (0.6, 2, 0.29 (Fig.2), 0.4, 4.25, 0.43 (Fig.3 where only one of the two L_j curves available here is shown). For $\beta < 0$ the L_j curves are symmetric to those presented in Figs.1-3. (All the L_j curves are symmetric in β because of the same symmetry of the function D_1 from (2.3)).

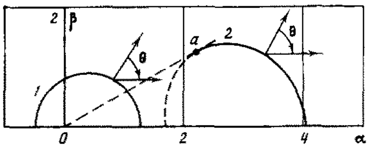

Fig.1

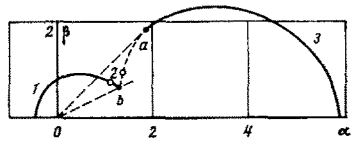

Fig.2

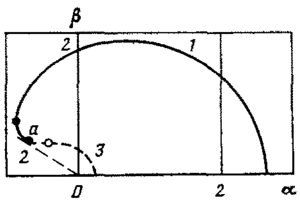

Fig.3

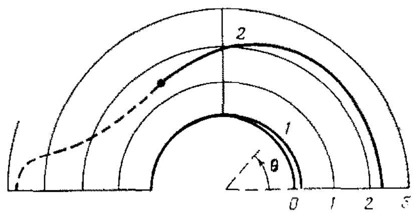

Fig.4

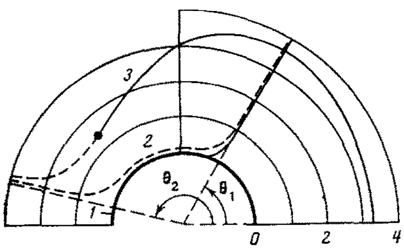

Fig.5

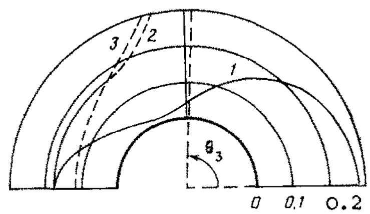

Fig.6

As follows from the preceding, for a fixed angle θ in the x_1, x_2 plane the stationary points on L_j are defined as points at which the external normal to the curve makes an angle θ with the positive direction of the α axis. The angle θ is here measured clockwise (Fig.1) if we have $c_n > 0$ on L_j, and counter-clockwise if $c_n < 0$. On all the curves shown in Figs.1-3, $c_n > 0$.

The curvature of the curves in Fig.1 never vanishes, consequently, for any angle θ there is just one point on L_j. Therefore, one wave of the form (4.4) corresponds to each curve here.

For $\beta > 0$ on the curve L_j in Fig.2 there are two points of inflection marked with open circles. At these points the normals make angles $\theta_1 \approx 58.8°$ and $\theta_2 \approx 166.4°$ with the positive direction of the axis α. Consequently, three waves generated by one curve will propagate in domains of the layer $\theta_1 < |\theta| < \theta_2$. Indeed, three stationary points correspond to such angles, and (4.4) is valid even in the case when $\varkappa_m < 0$, except $-\operatorname{sign} c_n^m$ must be substituted in place of $\operatorname{sign} c_n^m$. (The curvature $\varkappa$ at the point $(\alpha_*, \beta_*) \in L_j$ is considered to be negative if the curve L_j in the neighbourhood of this point is concave with respect to the internal domain bounded by it). One wave propagates in the remaining domain of the layer.

We have one point of inflection with an angle $\theta_3 \approx 87.7°$ for $\beta > 0$ in Fig.3. Here three waves propagate in the domain $|\theta| > \theta_3$, while one propagates in the domain $|\theta| < \theta_3$ $(|\theta| \leqslant \pi)$.

The curvature of the curves $\varkappa_m$ equals zero at the points of inflexion, while $\varkappa_m \to 0$ near these points. To calculate the field in these domains it is necessary to use formulas of the method of stationary phase for second-order stationary points and for closely located stationary points /9/.

We note that despite the decrease in the field in the neighbourhood of the singular directions as $r^{-1/2}$, the energy flux P through a cylindrical surface $r = \mathrm{const}, 0 \leqslant x_3 \leqslant 1$ will be bounded as $r \to \infty$ because of the smallness of the apex angle θ of the special domain where $\varkappa_m \to 0$ $(\theta \sim r^{-3/2})$.

There will be no domains with a different number of waves generated by one L_j curve propagating in the case of problem A_0, in the case of problem C_0 for a half-space, and for plane problems.

Graphs of the dependence of P_r on θ corresponding to L_j curves in Figs.1-3 are presented in Figs.4-6 for the same values of w, Ω, v_0. Readout of the values of P_r with factors $4\pi r$ is carried out from the circle enclosed by heavy lines. The curves or sections of L_j curves and their corresponding curves $4\pi r P_r(\theta)$ are marked with identical numbers.

It is seen that the greatest contribution to the energy flux P_r is made by sections of L_j curves with small curvature. Also singnificant is the contribution of such sections of the L_j curves where $\operatorname{grad} D_1$ is small (the direction $\theta = \pi$ in Fig.4, and the corresponding section of the second curve in Fig.1).

Finally, we note the following. Points of the L_j curves at which the tangents to the curves pass through the origin are denoted by the letters a in Figs.1 and 3 and a, b in Fig.2. The sections of the curves between these points and symmetric points to them with respect to α generate reverse waves, i.e., waves travelling from infinity. (for $\beta > 0$ these sections of the curves are shown dashed in Figs.1-3). There is no wave motion in r on the boundary of the propagation domain for such waves. However, the reverse waves carry energy here to infinity and yield no singularities in the energy flux distribution (the parts of the curves shown dashed in Figs.4-6). In these cases, the analogue to the Sommerfeld radiation principle is spoiled, i.e., the selection principle for waves travelling in a moving coordinate system from the source to infinity.

Therefore, here as in the plane problem /6/, for the case of pre-Rayleigh motion considered in this paper, the limiting absorption principle is equivalent to the energy principle, but is not equivalent to the analogue of the Sommerfeld radiation principle.

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